MATH 504 HOMEWORK 2

Due Friday, September 21.

Problem 1. *In ZF^- prove the Schröder-Bernstein theorem i.e. that if $A \preceq B$ and $B \preceq A$ implies that $A \approx B$.*
Hint: Suppose $f : A \to B$ and $g : B \to A$ are one-to-one.Set $A_0 = A$, $B_0 = B$, $A_{n+1} = g"B_n$, $B_{n+1} = f"A_n$, $A_\infty = \bigcap_n A_n$, $B_\infty = \bigcap_n B_n$. Let $h(x)$ be $f(x)$ if $x \in A_\infty \cup \bigcup_n (A_{2n} \setminus A_{2n+1})$. Otherwise let $h(x)$ be $g^{-1}(x)$. Show that h is well defined and $h : A \to B$ is one-to-one and onto.

Problem 2. *Assume CH (but not GCH). Show that for every natural number $n > 0$, $\omega_n^\omega = \omega_n$.*

Problem 3. *Show that for infinite cardinals $\kappa \geq \lambda$,*

$$|\{X \subset \kappa : |X| = \lambda\}| = \kappa^\lambda.$$

Problem 4. *Let κ be a regular uncountable cardinal, and $\langle C_\eta \mid \eta < \tau \rangle$ be a family of club subsets of κ for some $\tau < \kappa$. Prove that $\bigcap_{\eta<\tau} C_\eta$ is a club.*

Problem 5. *Let κ be a regular uncountable cardinal, and $f : \kappa \to \kappa$ be any function. Show that $\{\alpha < \kappa \mid (\forall \xi < \alpha)(f(\xi) < \alpha))\}$ is a club.*

Problem 6. *Let κ be the least inaccessible cardinal, such that κ is the κ-th inaccessible cardinal. Show that κ is not Mahlo. (Hint: Use $f(\lambda) = \alpha$ where λ is the α-th inaccessible cardinal.)*